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KEPLER'S PROBLEM.

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AT the end of the Fourth Part of his work "De Motibus Stellæ Martis," Kepler proposes the problem so long known by his name. His statement, translated from the Latin, is as follows:—

"But having given the mean anomaly, there is no geometrical method of arriving at its coequal, that is, the eccentric anomaly. For the mean anomaly is made up of two parts of area, a sector and a triangle, the former of which is counted by means of the eccentric arc, the latter by the sine of that arc multiplied into the value of the maximum triangle. But the proportions between the arcs and their sines are infinite in number. Therefore the sum of both being given it cannot be told how great the arc is, or how great is the sine of that arc corresponding to this sum, unless we first inquire how great an area is swept by the given arc, that is, unless we shall have constructed tables and afterwards shall have worked from them.

"This is my opinion of it; and by as much less as it seems to have of geometrical beauty, so much the more do I exhort geometers to solve for me this problem:

"*Given the area of a part of a semicircle, and given a point of the diameter, to find the arc and the angle at that point, the given area being included by the legs of this angle and by this arc; Or: To cut the area of a semi circle in a given ratio from any point whatsoever of the diameter.*

"I am sufficiently satisfied that it cannot be solved *a priori*, on account of the different nature of the arc and the sine. But if I am mistaken, and any one shall point out the way to me, he will be in my eyes the great Apollonius."

This is Kepler's opinion put forth nearly three centuries ago. The attempts to solve this famous problem, however, still continue, and nearly every year gives us two or three new solutions.—

The equation to be solved is the well known transcendental one,

$$E = M + e \sin E,$$

from which M and e being given we are to find E. The direct solution of this equation being impossible, the solutions that are given take a variety of forms, and for convenience we may arrange them in three classes:

(1). The pedagogic methods.

(2). The methods by series.

(3). The indirect method.

In the first class we put all the attempts to reduce the solution to a fixed set of rules, according to which nothing is left to the judgment of the computer, and where simple numerical accuracy, such as one might find in a child, or in a machine, is all that is required. This class will include many of the solutions that have been published, from that of Seth Ward, Professor at Oxford in 1649, to some of the most recent ones. The chief value of these solutions, and at the same time their weakness, is that they leave nothing uncertain in the process. They are therefore adapted to beginners in astronomy, and to those who always work by rule of thumb. The amount of intelligence required in the computer is a minimum.

The methods by series comprise the solutions given by Lagrange, Poisson, Bessel, Hansen, and other astronomers. These solutions are remarkable on account of the elegant and peculiar analysis employed, and they have an important use in theoretical astronomy. The series proceed according to powers of the eccentricity which are multiplied by the sines of multiples of the mean anomaly. The first systematic solution of this kind seems to have been given by Lagrange, who employed his well known theorem for determining the roots of all kinds of equations by means of series. This theorem, published in 1770, can be applied directly to Kepler's problem.

The indirect method, or solution by trial, is that, I think, to which every working astronomer finally comes. Of course, when the eccentricity is very small, or where tables of a planet have been computed, the equation of the center is tabulated, and this disposes of the matter once for all. But in case the eccentricity is as great as $\frac{2}{10}$ or $\frac{3}{10}$, and no tables are at hand, the indirect method does not require more than half the time of the other methods, and its accuracy is as great as one pleases. The advantage of this method consists in the indeterminate form in which the solution is left, and in the opportunity that is thus given for the intelligence and skill of the computer to come into use. Now this is the point that writers who are continually bringing forward solutions of the first kind fail to see; and at the same time they seem to be shocked at the notion of making a good guess in solving an equation. But certainly it is not necessary to be stupid in order to be mathematical; and there are so many problems to be solved that we need not fear of taking an undue advantage of things, or that our ingenuity will ever grow dull for lack of opportunity.